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| Mathematics Design Collaborative |
| State of Georgia Department of Education |



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| Understanding Standard Deviation |
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| *What’s Spread Got to do with it?* |
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| INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON |

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| **MATHEMATICAL GOALS** |
| This lesson unit is intended to help you assess how well students are able to: |
| * Interpret the meaning of standard deviation as a measure of the spread away from the mean of a set of data. * Compare data sets using standard deviation |

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| **GEORGIA STANDARDS OF EXCELLENCE** |
| This lesson also asks students to select and apply mathematical content from across the grades, including the *content standards*:   * **MGSE9-12.S.ID.2** **Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, ~~mean absolute deviation~~, standard deviation) of two or more different data sets.** * S.IC Make inferences and justify conclusions from sample surveys, experiments, and observational studies   This lesson involves a range of *mathematical practices* from the standards, with emphasis on:   * SMP2. Reason Abstractly * SMP3. Construct viable arguments and critique the reasoning of others * SMP7. Look for and make use of structure |
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| **INTRODUCTION** |

This lesson is structured in the following way:

* Before the lesson, students work individually to complete a task that is designed to reveal their current understandings and difficulties. You should then review their work and create feedback questions that will help them improve their solutions.
* At the start of the lesson, students will work in pairs to arrange a set of graphs in order from least standard deviation to greatest. They will then order a set of binomial expression cards. Finally they will arrange a set of situation cards.
* After a whole class discussion, students will work individually on the post assessment. This is identical to the pre-assessment, allowing students to demonstrate the progress they have made during the lesson.

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| **MATERIALS REQUIRED** |
| * Each individual student will need a copy of the pre-assessment and a copy of the post-assessment. * Each small group of students will need a set of cards with graphs, data sets and situations. * Each small group may also use a large piece of paper to display their sequences of cards. |
| **TEACHER PREP REQUIRED** |
| Teacher, be advised that prior to the lesson, the following preparations/copies will need to be made: |
| * Be sure to allow adequate time to cut out the cards for the card sort before the lesson. |

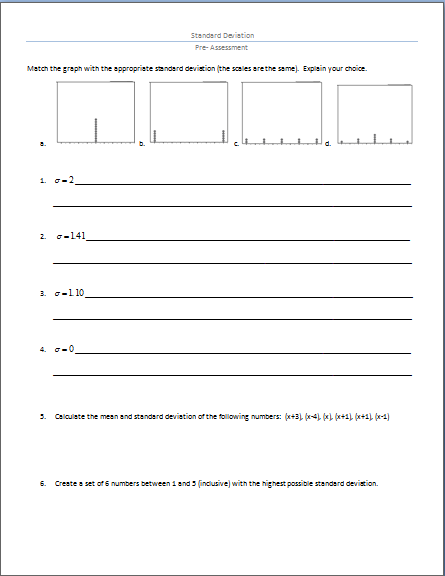
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| **TIME NEEDED:** | | | | | | | |
| For Pre-Assessment: | 15 minutes | For Lesson: | 85 minutes | For Post: | 15 minutes |  |
| Special Note(s) about timing: Timings are approximate and should be adjusted based on the needs of the students. | | | | | | | |

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| **FRAMING FOR THE TEACHER:** |
| This Formative Assessment Lesson is intended to challenge a student’s understanding of standard deviation. Rather than focus on computational skills, this lesson attempts to get to the concept of standard deviation as a measure of spread about the mean. |

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| **FRAMING FOR THE STUDENTS:** |
| Say to the students: |
| *This activity will take about two days for us to complete.* |
| *The reason we are doing this is to be sure that you understand standard deviation before we move on to a new idea.* |
| *You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!* |

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| PRE-ASSESSMENT |

Have the students do this task in class or for homework a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the lesson.



Ask students to work through the task.

*Spend fifteen minutes working individually on this task. Don’t worry if you can’t understand and do everything. There will be a lesson tomorrow that should help you understand the math better. By the end of that lesson your goal is to answer the questions with confidence.*

Students should do their best to answer these questions, without teacher assistance. It is important that students are allowed to answer the questions on their own so that the results show what students truly do not understand.

Students should not worry too much if they cannot understand or do everything on the pre-assessment, because in the next lesson they will engage in a task which is designed to help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently.

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| COLLABORATION TIME/READING STUDENTS RESPONSES |
| **You Will Not “Grade” These!** |

Collect students’ responses to the task. Make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of issues that will arise during the lesson itself, so that you may prepare carefully.

We suggest that you do not score students’ work. Research suggests this will be counterproductive, as it encourages students to compare their scores, and distracts their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page.

We suggest that you write a list of your own questions, based on your students’ work, using the ideas that follow. You may choose to write questions on each student’s work. If you do not have time to do this, just select a few questions that will be of help to the majority of students. These can be written on the board during the plenary discussion and be left visible throughout the lesson.

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| **GUIDING QUESTIONS *Here are some questions that you may find useful as you address misconceptions in class:*** | |
| ***COMMON ISSUES*** | ***SUGGESTED QUESTIONS AND PROMPTS*** |
| **Student has difficulty getting started**  For example: The questions are mostly unanswered. | * *Do you need exact numbers to compare standard deviation?* * *What is standard deviation a measure of?* |
| **Students does not explain or misinterprets**  For example: The student explains that the more spread out the data, the smaller the standard deviation | * *What kind of data set has a standard deviation of zero?* * *What kind of data set has a standard deviation of 10?* |
| **Student calculates the mean or standard deviation wrong.**  For example: The student doesn’t take the square root after finding the average of the deviations. | * *What does the formula for mean look like?* * *What does the formula for standard deviation look like?* |
| **Student has a data set that does not have the highest possible standard deviation** | * *How could you spread these numbers out even more?* * *What does “spread out” mean in math terms?* |

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| LESSON | |
| **SUGGESTED LESSON OUTLINE:** | |
| **Part 1: Whole-Class Introduction:** | **Time to Allot: (10 minutes)** |
| Have students arranged in pairs. Each student should have a mini whiteboard and a marker. Frame the lesson for them in this way:   * *Today we are going to investigate the idea of standard deviation more closely. We will look at it more conceptually instead of solely as a calculation of numbers.* * *What is standard deviation a measure of?* * *Using the numbers 1-100, create a data set of 4 numbers that has the smallest standard deviation possible*   Have the students show their work and compare the answers. Call on students with different answers and have them justify their answers. Try to help students realize that there are many different solutions to this question.   * *Now using the numbers 1-100, create a data set of 4 numbers that has the largest standard deviation possible.*   Have the students show their work and compare answers. Call on students with different answers and have them justify their answers. Try to help students realize that there is only one solution to this question.   * *Now you are going to work on a collaborative activity in which you will arrange different data sets according to their standard deviation. Remember what you have learned to help you with this task.* | |
| **Part 2: Collaborative Activity:** | **Time to Allot: (60 minutes)** |
| Put students into their pairs according to your analysis of student errors. | |
| **Working in pairs part 1(15 minutes)**  Organize students into groups of two. Provide each group with a set of 5 graph cards.  Help students understand the task and the instructions   * *Your task is to place the 5 graphs in order from smallest standard deviation to largest standard deviation* * *Take turns with your partner positioning graphs until all the graphs have been placed.* * *Explain to your partner why you placed the graph where you did.* * *When you agree on the order of your graphs, compare your answers with a neighboring group.*   Once the students have settled on an order, distribute the variable expressions cards. The students will match them to the graphs from the first set of cards. Students should realize that the expressions represent numbers and that the set does have a standard deviation.   * *Now match the variable expressions with the appropriate graph.* * *Calculate the standard deviation of each of the variable expressions*. * *Use the standard deviations to check your order*.   **Support student problem solving OR Support students’ thinking**  Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions that help students to clarify their thinking. You may discover that some students experience some difficulty communicating their ideas, but be patient and encourage them to think it out.  **Plenary whole-class discussion (Collaborative Activity) (10 minutes)**  After the groups have had time to check their responses, engage the students in a whole class discussion about what they learned.   * *How did you get started?* * *Did any results surprise you?* * *Did you notice any patterns?* * *How did you distinguish between the middle three?* * *Could you make a graph that is somewhere between any other two graphs?* * *What can we say about standard deviation, based on what you saw with the graphs?* * *What would happen if you changed the scale of the graphs? Do the numbers even matter?*   Remember, the goal is to get the students to see that the further the points are away from the mean, the larger the standard deviation, and the closer the points are to the mean, the smaller the standard deviation.  **Working in pairs part 2 (15 minutes)**  Now distribute the scenario cards to each group.   * *Now take the scenario cards and put them in order from least to greatest according to standard deviation.* * *Take turns with your partner positioning scenarios until all the cards have been placed.* * *Explain to your partner why you placed the card where you did.* * *When you agree on the order of your cards, compare your answers with a neighboring group.* | |
| **Part 3: Plenary (Summary) Discussion:** | **Time to Allot: ( 15 minutes)** |
| NOTE: *“Scribing” helps to increase student buy-in and participation. When a student answers your question, write the student’s name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!*  **Plenary whole-class discussion**  Once the students have placed their cards, engage the class in a whole class discussion.   * *How did you begin?* * *What were important characteristics that helped you place cards?* * *Which card did you place first? Why?* * *Which card was the hardest to place?* * *Did the size of the numbers have any effect on your placement?* * *How did you estimate the standard deviation without knowing numbers in some cases?* * *Which characteristics affected the standard deviation the most?* | |
| **Part 4: Improving Solutions to the Assessment** | **Time to Allot: ( 15 minutes)** |
| The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment (which is sometimes the exact same as the Pre-Assessment). In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no “X” mark means that it must have been right. Until students become accustomed to UNGRADED FORMATIVE assessments, they may naturally do this. Teachers often report success by handing students a list of the guiding questions to keep in mind while they improve their solutions.  This is an opportunity for students to improve upon their earlier work, based on what they have learned through the collaborative activity.  \*If you are running out of time, you could set this task in the next lesson or for homework.  Give students back their work on the pre-assessment task along with a copy of the post-assessment.   * *Work on your own for fifteen minutes.* * *I’m giving you your own answers, and a new sheet to work on.* * *Read through your original solution and think about what you learned during the lesson.* * *I want you to use what you learned to improve your solution.* | |
| PRE-ASSESSMENT (Answer Key) | |
| *What’s Spread Got To Do With It? Pre-Assessment Name: \_\_\_\_\_\_\_\_****Key****\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*  Match the graph with the appropriate standard deviation (the scales are the same). Explain your choice.   1. b.  c. d. 2. \_\_\_\_**b**. **This standard deviation matches the graph with the largest spread, meaning that the numbers are as far away from the mean as possible.**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 3. \_\_\_\_**c. These points are spread out equally so they are making the spread about the mean larger.**   \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   1. \_\_\_\_**d. Because standard deviation measures the average distance from the mean, the closer the points are to the mean, the smaller the standard deviation is. Because this graph has 4 points on the mean, they would be contributing zero to the standard deviation, making it smaller.**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 2. \_\_\_\_\_**a. This standard deviation matches a graph with no spread. In other words, a graph in which all the numbers are the same.**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 3. Calculate the mean and standard deviation of the following numbers: (x+3), (x-4), (x), (x+1), (x+1), (x-1)   **The mean is x, the standard deviation is the square root of 28.**   1. Create a set of 6 numbers between 1 and 5 (inclusive) with the highest possible standard deviation.   **The set of numbers is 1,1,1,5,5,5** | |

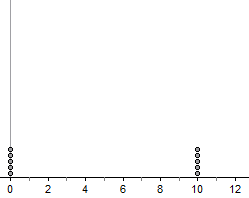
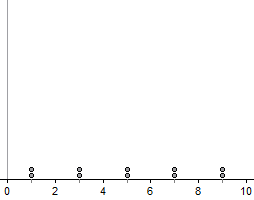
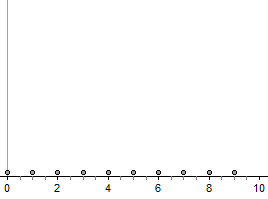
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| Collaborative Activity (Answer Key) |
| **Working in pairs part 1 Cards**  Place the cards in order from least to greatest according to their standard deviation. *Do not calculate the standard deviation!* Once you and your partner agree on an order, calculate the standard deviation and check how you did.  **Key (ranked from least (1) to greatest (5) )**  (**5) (SD = 5) (4) (SD = 2.8723)**     1. **(SD = 0) (3) (SD = 2.8284)**      1. **(SD = 1.6733)** |

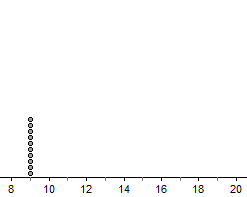
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| PRE-ASSESSMENT |
| *What’s Spread Got To Do With It? Pre-Assessment Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*  Match the graph with the appropriate standard deviation (the scales are the same). Explain your choice.   1. b.  c. d. 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_   1. Calculate the mean and standard deviation of the following numbers: (x+3), (x-4), (x), (x+1), (x+1), (x-1) 2. Create a set of 6 numbers between 1 and 5 (inclusive) with the highest possible standard deviation. |

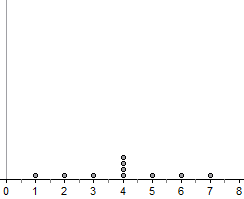
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| COLLABORATIVE ACTIVITY |

**Working in pairs part 1 Cards**

Place the cards in order from least to greatest according to their standard deviation. *Do not calculate the standard deviation!* Once you and your partner agree on an order, calculate the standard deviation and check how you did.







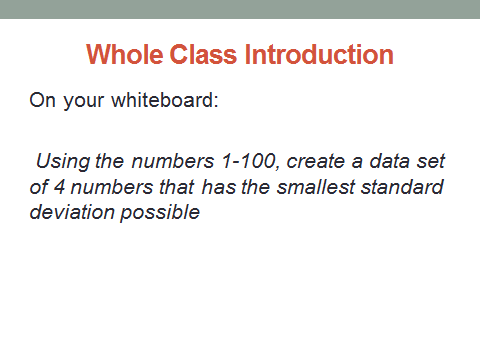
**Variable Expression Cards**

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| x-5, x-5, x-5, x-5, x-5, x+5, x+5, x+5, x+5, x+5 |
| x-4.5, x-3.5, x-2.5, x-1.5, x-0.5, x+0.5, x+1.5, x+2.5, x+3.5, x+4.5 |
| x, x, x, x, x, x, x, x, x |
| x-4, x-4, x-2, x-2, x, x, x+2, x+2, x+4, x+4 |
| x-3, x-2, x-1, x, x, x, x, x+1, x+2, x+3 |

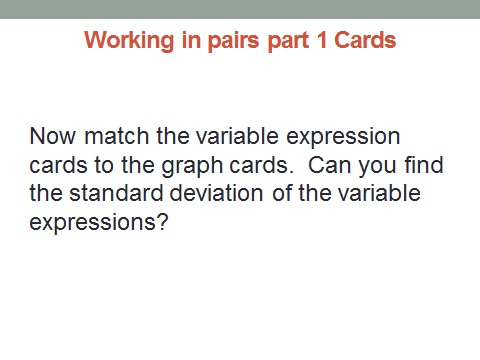
**Working in Pairs Part 2 (Scenario Cards)**

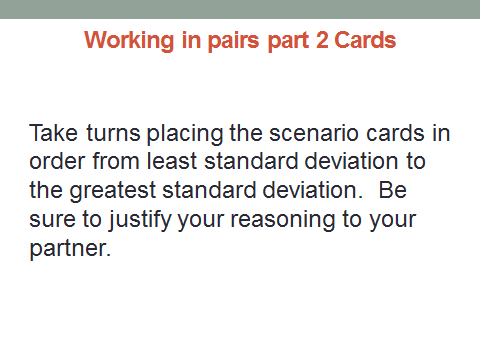
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| The ages of students in a kindergarten classroom |
| The ages of students in a 10th grade Math class |
| The ages of students in a small school system |
| The ages of students in a large school system |
| The ages of students on their 16th birthday. |
| The ages of students in a high school |
| The ages of teachers in a high school |
| The ages of students in a college |

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| **Whole Class Introduction** |

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| **Collaborative Activity Instructions:** |
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| **Collaborative Plenary Discussion Questions:** |
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